Soft Collinear Effective Theory

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Outline

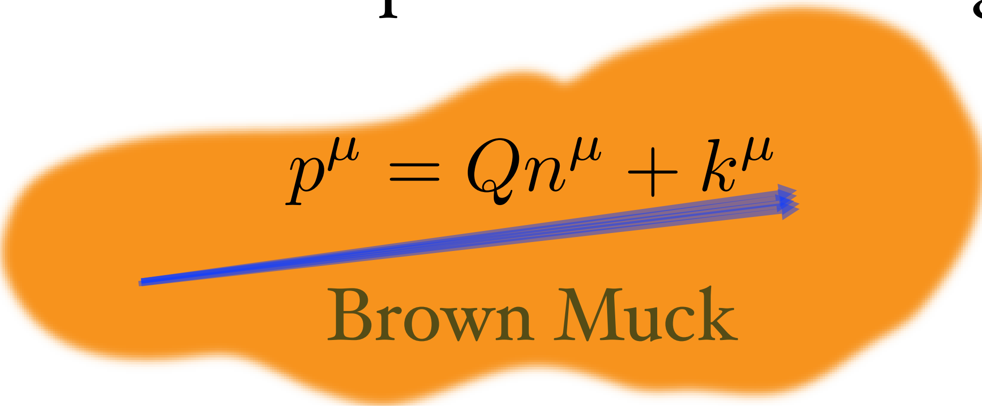
- Soft Collinear Effective Theory: Overview of Formalism
  - Structure of theory
  - Factorization
- Applications
  - Color-suppressed $B \to D\pi$ decays
  - Radiative Upsilon decays
  - $B \to \pi + \ell\bar{\nu}$
- Conclusion

Soft Collinear Effective Theory: Overview

Soft-Collinear Effective Theory: Overview

Bauer, Fleming, Luke, Pirjol, Stewart

- SCET: Effective theory of highly energetic, approximately massless particles interacting with a soft background



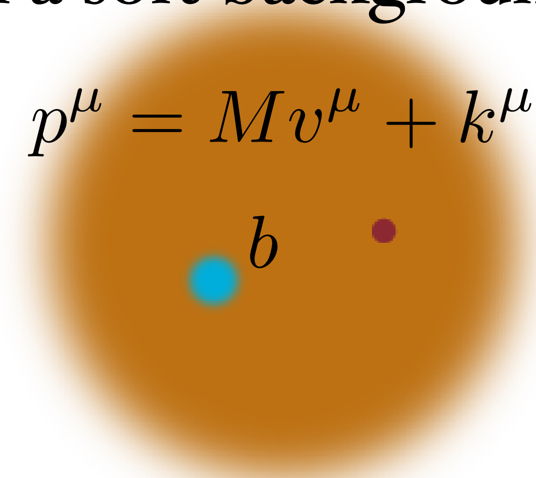
Energetic: $Q \gg \Lambda_{\rm QCD}$

Light-like: $n^\mu = (1,0,0,1)$

Residual Momentum: $k \sim \Lambda_{\rm QCD}$

Expansion in: $\lambda \sim \Lambda_{\rm QCD}/Q$

- HQET: Effective theory of very massive particle interacting with a soft background



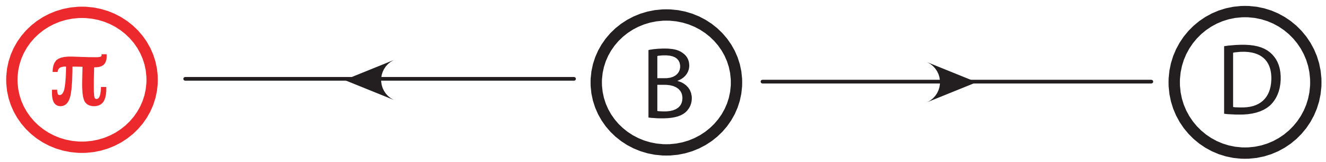
Heavy: $M \gg \Lambda_{\rm QCD}$

Static: $v^\mu = (1,0,0,0)$

Residual Momentum: $k \sim \Lambda_{\rm QCD}$

Expansion in: $\Lambda_{\rm QCD}/M$

Example



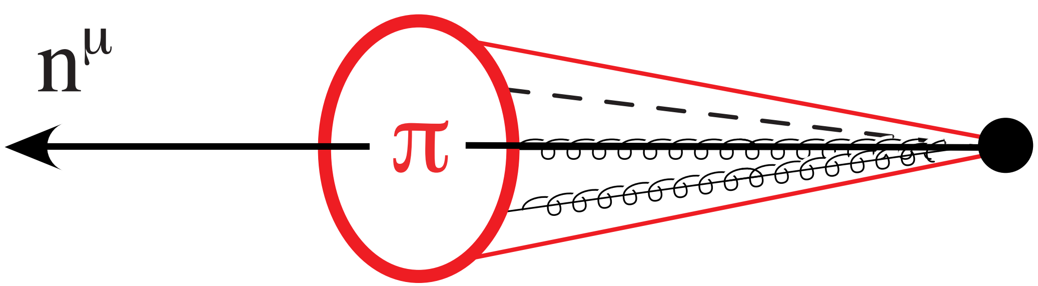
Pion has: $\color{red}{p_\pi^\mu} \;=\; (2.3\,\mathrm{GeV})n^\mu = Q\,n^\mu \qquad n^2 = \bar{n}^2 = 0,\;\; (\bar{n}\cdot p = p^-)$

Soft constituents:

$$p_s^\mu = (p^+, p^-, p^\perp) \sim (\Lambda, \Lambda, \Lambda)$$

Collinear constituents:

$$p_c^\mu = (p^+, p^-, p^\perp) \sim \left(\frac{\Lambda^2}{Q}, Q, \Lambda\right) \sim Q(\lambda^2, 1, \lambda) \qquad \lambda = \frac{\Lambda}{Q}$$



Degrees of freedom in SCET

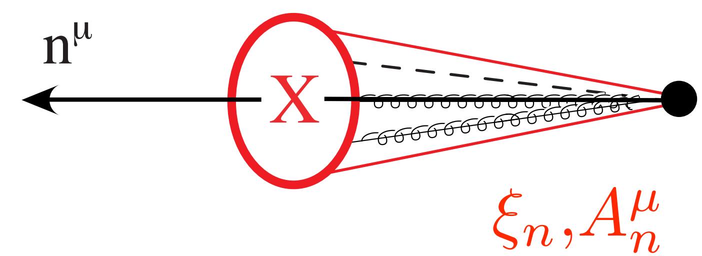
Introduce fields for infrared degrees of freedom (in operators)

modes	$p^\mu = (+, -, \perp)$	p^2	fields
collinear	$Q(\lambda^2, 1, \lambda)$	$Q^2\lambda^2$	$\xi_n,\ A_n^\mu$
soft	$Q(\lambda, \lambda, \lambda)$	$Q^2\lambda^2$	$q_s,\ A_s^\mu$
usoft	$Q(\lambda^2, \lambda^2, \lambda^2)$	$Q^2\lambda^4$	$q_{us},\ A_{us}^\mu$

$\mathrm{SCET_I}$ → Energetic jets $\quad \Lambda^2 \ll Q\Lambda \ll Q^2$

usoft $\quad p^\mu \sim \Lambda$

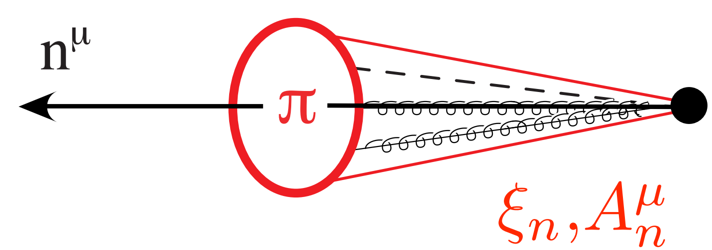
collinear $\quad p_c^2 \sim Q\Lambda, \quad \lambda = \sqrt{\Lambda/Q}$



$\mathrm{SCET_{II}}$ → Energetic hadrons

soft $\quad p^\mu \sim \Lambda$

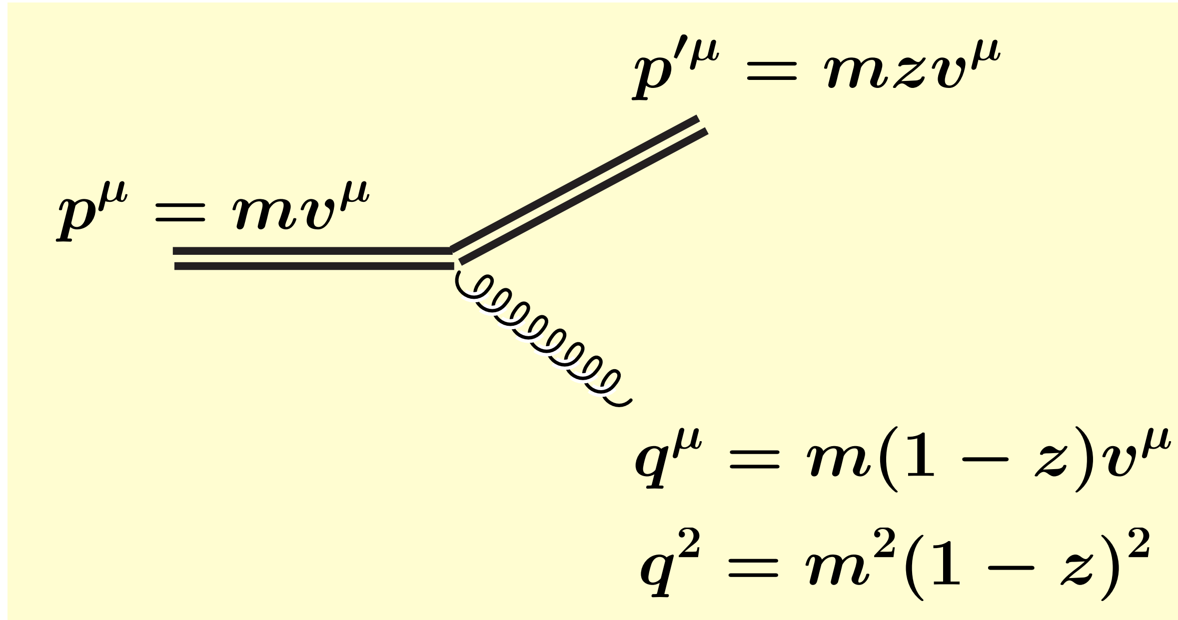
collinear $\quad p_c^2 \sim \Lambda^2, \quad \lambda = \Lambda/Q$



Soft-Collinear Effective Theory: Overview

HQET

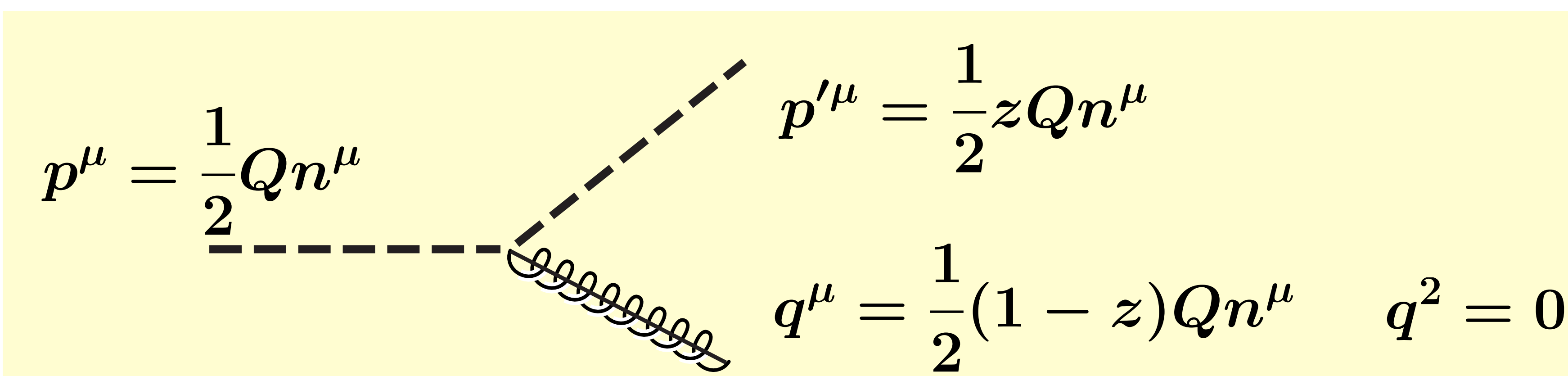
Not Allowed!!!



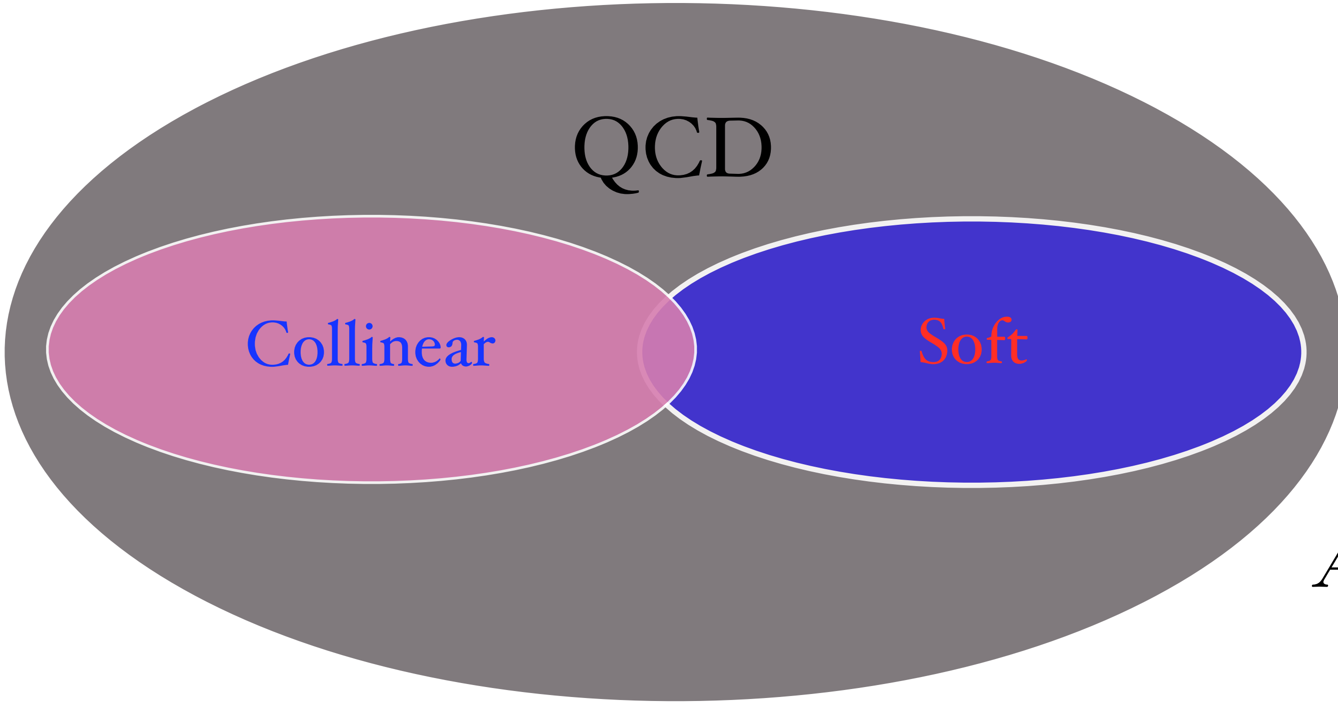
- Analogy with HQET breaks down:

SCET

O.K.



SCET Lagrangian



$$\psi(x) \to \psi_s(x) + \xi_n(x)$$

$$A^\mu(x) \to A^\mu_s(x) + A^\mu_n(x)$$

$$\mathcal{L}_c = \bar{\xi}_n \left\{ in\cdot D_c + i\not{D}_c^\perp \frac{1}{i\bar{n}\cdot D_c} i\not{D}_c^\perp + gn\cdot A_s \right\} \frac{\bar{\not{n}}}{2} \xi_n$$

$$\mathcal{L}_s = \bar{\psi}_s \; i\not{D}_s \psi_s$$

- Collinear sector: QCD in boosted frame
- Soft sector: QCD
- Coupled through a single term

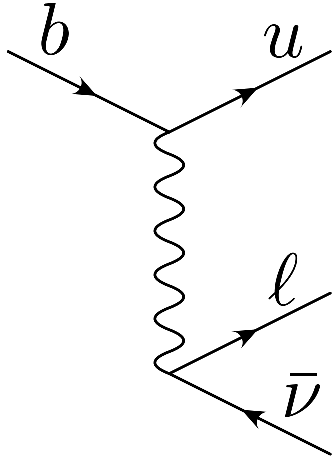
Symmetries & Properties

- Separate collinear and soft gauge symmetries
  - Powerful restriction on the form of operators allowed
  - Soft fields act as a background field to collinear fields
  - Any gauge symmetry connecting soft to collinear introduces a large scale
- Factorization of hard scale, Q, automatic
- Factorization of soft and collinear through field redefinition
- Global $U(1)$ helicity spin symmetry
- Reparameterization invariance which is a consequence of Lorentz invariance of QCD: Relates operators

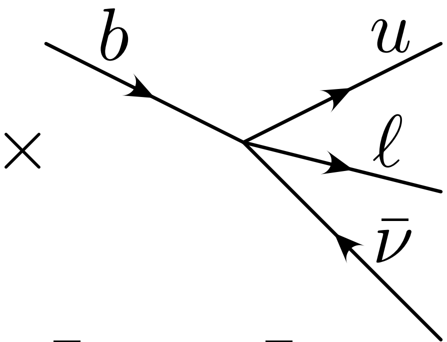
Factorization

Example: $b \to u + \ell\bar{\nu}$

- Integrate out the W boson to obtain Fermi theory

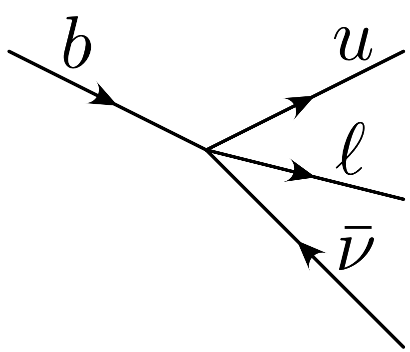


$$\longrightarrow \quad H_{ew}(1/m_W) \quad \times$$

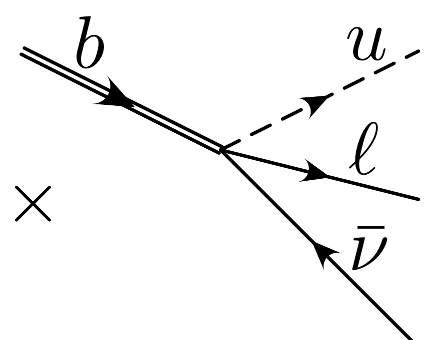


$$\bar{\psi}_u \Gamma \psi_b \bar{\psi}_\ell \Gamma \psi_\nu$$

- Integrate out the b-qaurk mass: HQET + SCET



$$\longrightarrow \quad H_b(1/m_b) \times$$



$$\bar{\psi}_u \Gamma \psi_b \bar{\psi}_\ell \Gamma \psi_\nu \quad \longrightarrow \quad H_b(1/m_b) \times \bar{\xi}_n \Gamma h_v \bar{\psi}_\ell \Gamma \psi_\nu \quad ???$$

Example: $b \to u + \ell\bar{\nu}$

- There is a problem!
  - Recal: separate collinear and soft gauge symmetries in SCET

Soft: both $\bar{\xi}_n$ and h_v transform in such a way that

$$\bar{\xi}_n \Gamma h_v \bar{\psi}_\ell \Gamma \psi_\nu \longrightarrow \bar{\xi}_n \Gamma h_v \bar{\psi}_\ell \Gamma \psi_\nu$$ Gauge Invariant

Collinear: only $\bar{\xi}_n$ transforms $\bar{\xi}_n \Gamma h_v \bar{\psi}_\ell \Gamma \psi_\nu$ ~~Gauge Invariant~~

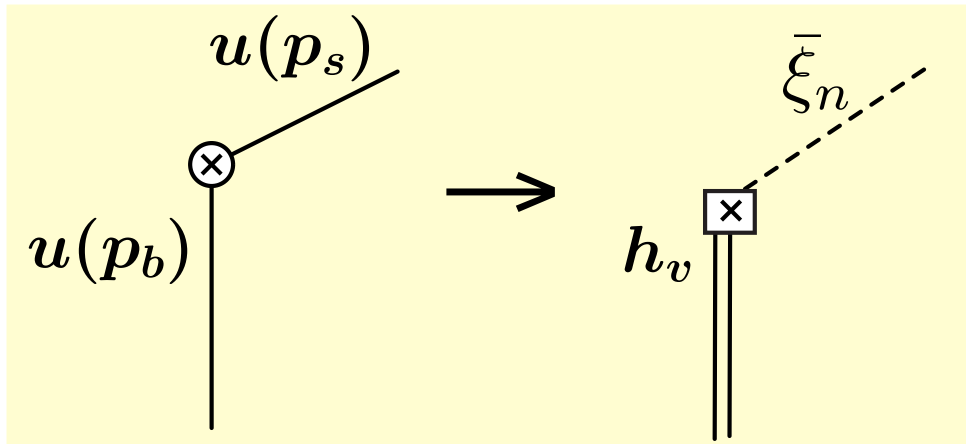
What's missing?!?!?!

Collinear Wilson Line $W = P \exp\left(ig \int_{-\infty}^{y} ds\, \bar{n}\cdot A_n(s\bar{n}^\mu)\right)$

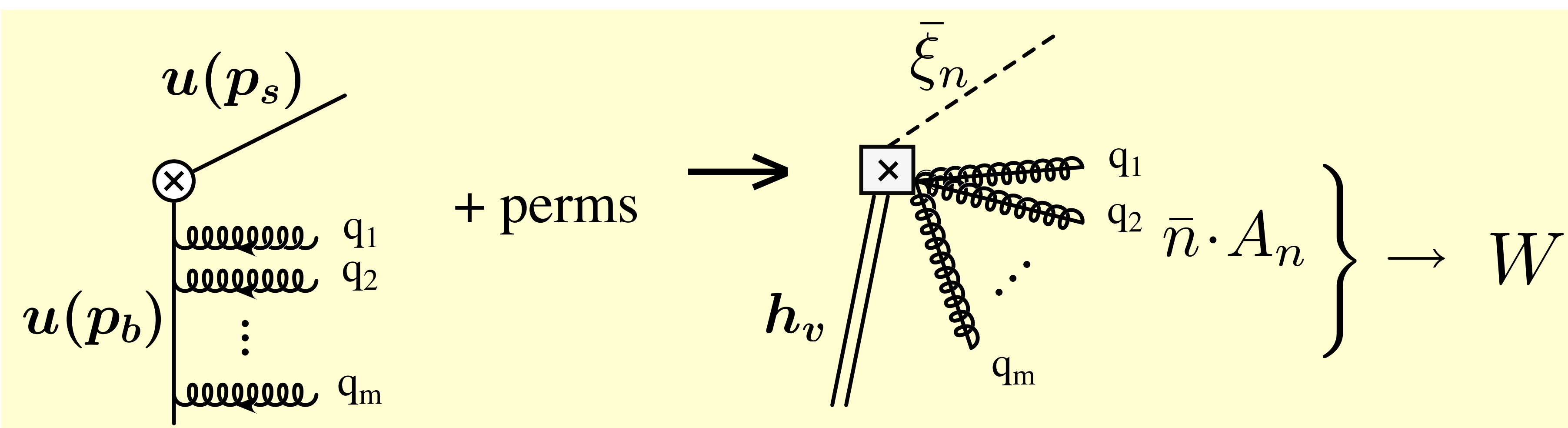
Collinear Wilson Line

Perturbative origin of the collinear Wilson line

- Leading order in α_s



- Higher orders



$$\bar{\xi}_n W \Gamma h_v \bar{\psi}_\ell \Gamma \psi_\nu$$

Factorization

- Hard factorization:

$$H_{ew}(1/m_W)H_b(1/m_b)\bar{\xi}_n W \Gamma h_v \bar{\psi}_\ell \Gamma \psi_\nu$$

- Collinear/Soft factorizaton:

  - Decouple Soft from Collinear in the Lagrangian

1) Soft Wilson Line $\quad Y(x) = \mathrm{P}\exp\left(ig\int_{-\infty}^{x} ds\, n\cdot A_s(ns)\right)$

2) Field Redefinition $\quad \xi_n(x) = Y(x)\xi_n^{(0)}(x)$

$$\mathcal{L}_c \to \bar{\xi}_n\left\{in\cdot D_c + i\not{D}_c^{\perp}\frac{1}{i\bar{n}\cdot D_c}i\not{D}_c^{\perp}\right\}\frac{\not{\bar{n}}}{2}\xi_n$$

- Factored Vertex: $\bar{\xi}_n W\Gamma Y^\dagger h_v\bar{\psi}_\ell\Gamma\psi_\nu$

Recap

What have we learned:

- SCET: EFT of collinear d.o.f. coupled to soft d.o.f.
  - Powerful gauge symmetries constrain operators
  - Decoupling via field redefinition

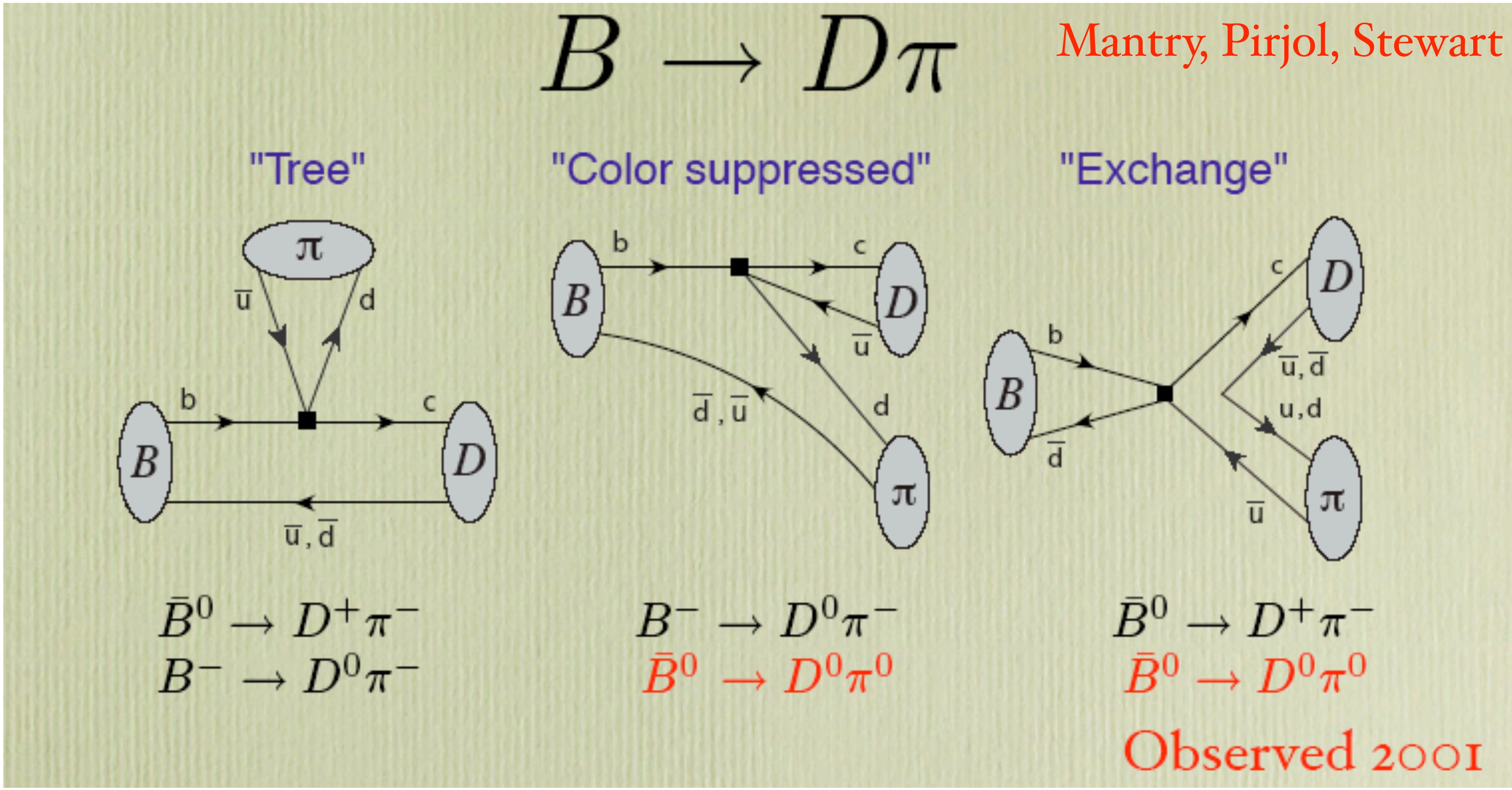
What is it good for?

- SCET is useful for understanding:
  - Factorization: Obtained from field redefinition and simple algebraic manipulations
  - Summation of Logarithms at the edges of phase space: Obtained from Renormalization Group Equations (RGEs)
  - Systematic Power Corrections in λ : Turn the crank

Applications

Color suppressed $B \to D\pi$ decays

Color-Suppressed



N_c^0 $\qquad$ $\dfrac{1}{N_c}$ $\qquad$ $\dfrac{1}{N_c}$

Data

(Cleo, Belle, Babar)

Type	Decay	Br(10^{-3})	Decay	Br(10^{-3})
I	$\bar{B}^0 \to D^+\pi^-$	2.68 ± 0.29	$\bar{B}^0 \to D^{*+}\pi^-$	2.76 ± 0.21
III	$B^- \to D^0\pi^-$	4.97 ± 0.38	$B^- \to D^{*0}\pi^-$	4.6 ± 0.4
II	$\bar{B}^0 \to D^0\pi^0$	0.29 ± 0.03	$\bar{B}^0 \to D^{*0}\pi^0$	0.26 ± 0.05
I	$\bar{B}^0 \to D^+\rho^-$	7.8 ± 1.4	$\bar{B}^0 \to D^{*+}\rho^-$	6.8 ± 1.0
III	$B^- \to D^0\rho^-$	13.4 ± 1.8	$B^- \to D^{*0}\rho^-$	9.8 ± 1.8
II	$\bar{B}^0 \to D^0\rho^0$	0.29 ± 0.11	$\bar{B}^0 \to D^{*0}\rho^0$	< 0.56

- Color- Suppressed decays are indeed suppressed

But

Large N_c is not very predictive

- How about using SCET & HQET?

Color Suppressed Decays in SCET

- Possible to derive a factorization formula in SCET

SCET operators are <u>power</u> suppressed in addition to being <u>color</u> suppressed

$$\lambda \sim \frac{\Lambda_{\rm QCD}}{E_\pi} \sim 0.2$$

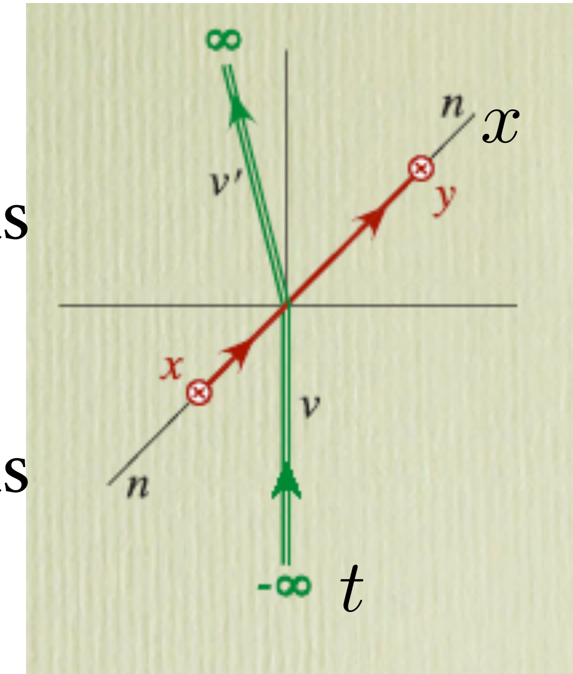
$$A_{00}^{D^{(*)}} = N_0^{(*)} \int dx\, dz\, dk_1^+\, dk_2^+\, \underbrace{T^{(i)}(z)}_{Q^2} \gg \underbrace{J^{(i)}(z, x, k_1^+, k_2^+)}_{Q\Lambda} \gg \underbrace{S^{(i)}(k_1^+, k_2^+)\, \phi_M(x)}_{\Lambda^2}$$

$$+A_{\rm long}^{D^{(*)}M}$$

- New non-perturbative function: $S^{(i)}(k_1^+, k_2^+)$

Color Suppressed Decays in SCET

- $S^{(i)}(k_1^+, k_2^+) = \langle D^{(*)}|O_s|B\rangle$ is the lightcone distribution function for the spectator quarks in the B and D



- It is universal for a particular set of directions $\{v, v', n\}$
  - Will be the same for D and D^*

- It is a complex function: large strong phases are natural

Comparison to Date

- Universality for D and D^*
  - Branching ratio
    $$Br(D^0\pi^0) = (0.29 \pm 0.03) \times 10^{-3}$$
    $$Br(D^{*0}\pi^0) = (0.26 \pm 0.05) \times 10^{-3}$$
  - Strong Phase
    $$\delta(D\pi) = 30.4 \pm 4.8^\circ$$
    $$\delta(D^*\pi) = 31.0 \pm 5.0^\circ$$
- Prediction
  $$r_{00}^{\rho} = \frac{A(\bar{B}^0 \to D^{*0}\rho^0)}{A(\bar{B}^0 \to D^0\rho^0)} = 1$$
- Can explain data
  $$|r^{D\pi}| = \frac{|A(\bar{B}^0 \to D^+\pi^-)|}{|A(B^- \to D^0\pi^-)|} = 0.77 \pm 0.05\,, \qquad |r^{D\rho}| = 0.80 \pm 0.09$$

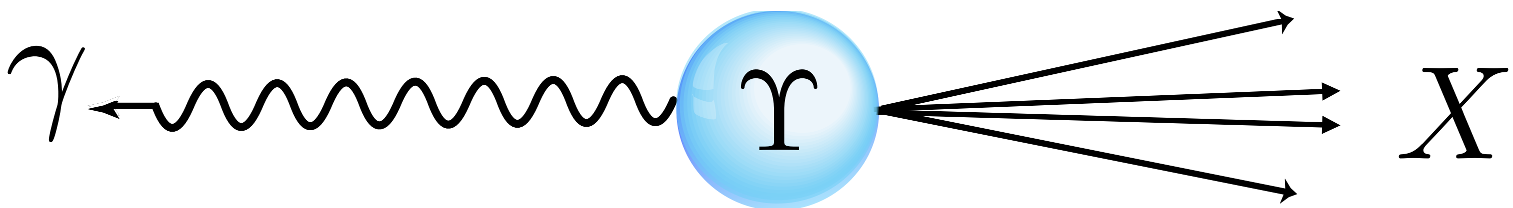
SCET Predicts
$$r^{DM} = 1 - \frac{16\pi\alpha_s m_D}{9(m_B + m_D)} \frac{\langle x^{-1}\rangle_M}{\xi(w_{max})} \frac{s_{\rm eff}}{E_M}$$

Natural sized parameter fits the data: $s_{\rm eff} \simeq (430\,\text{MeV})e^{i44^\circ}$

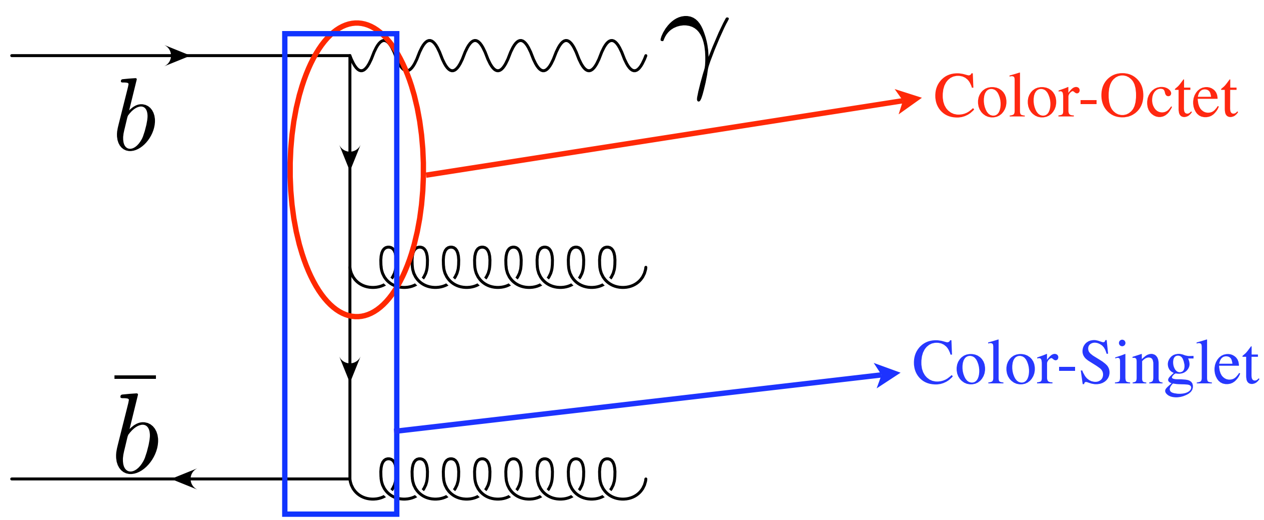
Radiative Υ Decay

Radiative Υ Decay

C.W. Bauer, SF, C.W. Chiang, A. Leibovich, I. Low,Phys.Rev.D64:114014,2001
SF & A. Leibovich, Phys.Rev.D67:074035,2003
SF & A. Leibovich, Phys.Rev.D70:094016,2004

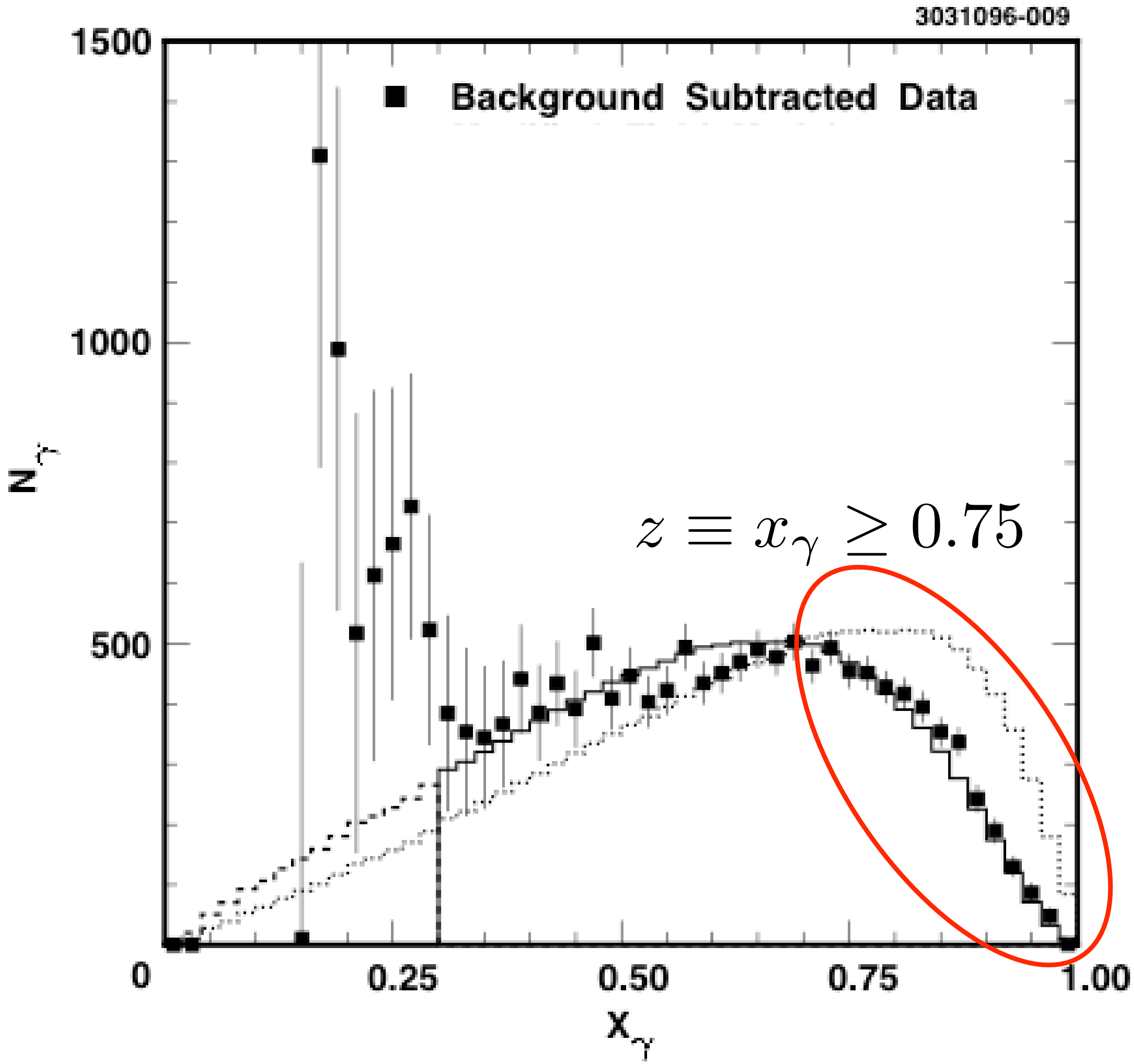


Short-distance process



- Differential decay rate measured: $d\Gamma/dz$ $\quad z \equiv x_\gamma \equiv 2E_\gamma/M_\Upsilon$

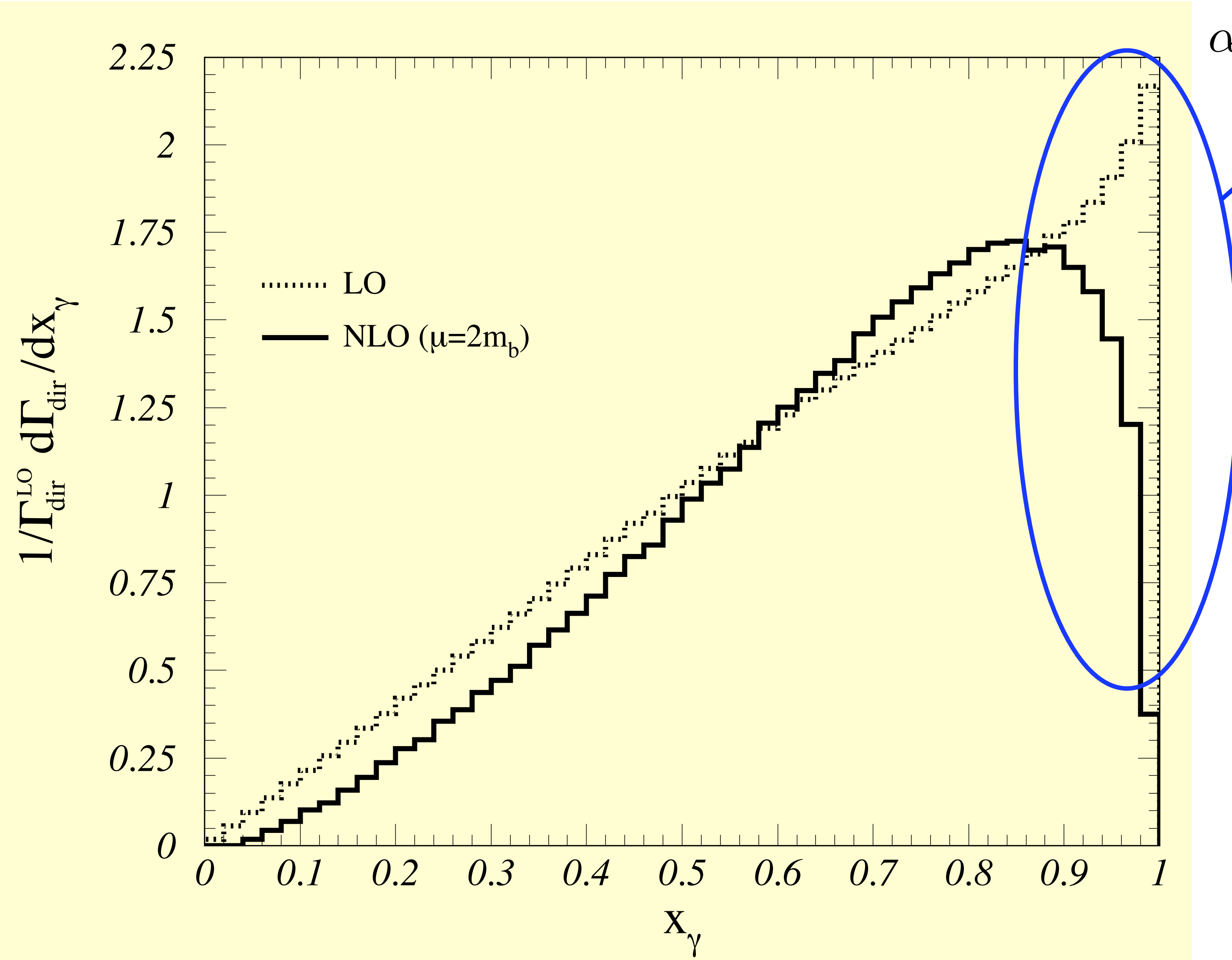
Radiative Υ Decay



Radiative Υ Decay

- Next-to-leading order calculation:

  M. Kramer, Phys. Rev. D60, 111503 (1999)



$\alpha_s(M_\Upsilon)\log(1-X_\gamma)\sim 1$

Large Logarithms

$z \equiv x_\gamma \geq 0.8$

Radiative Υ Decay

Use SCET in the endpoint region: $z \equiv x_\gamma \geq 0.7$

- Color-Octet Contribution:

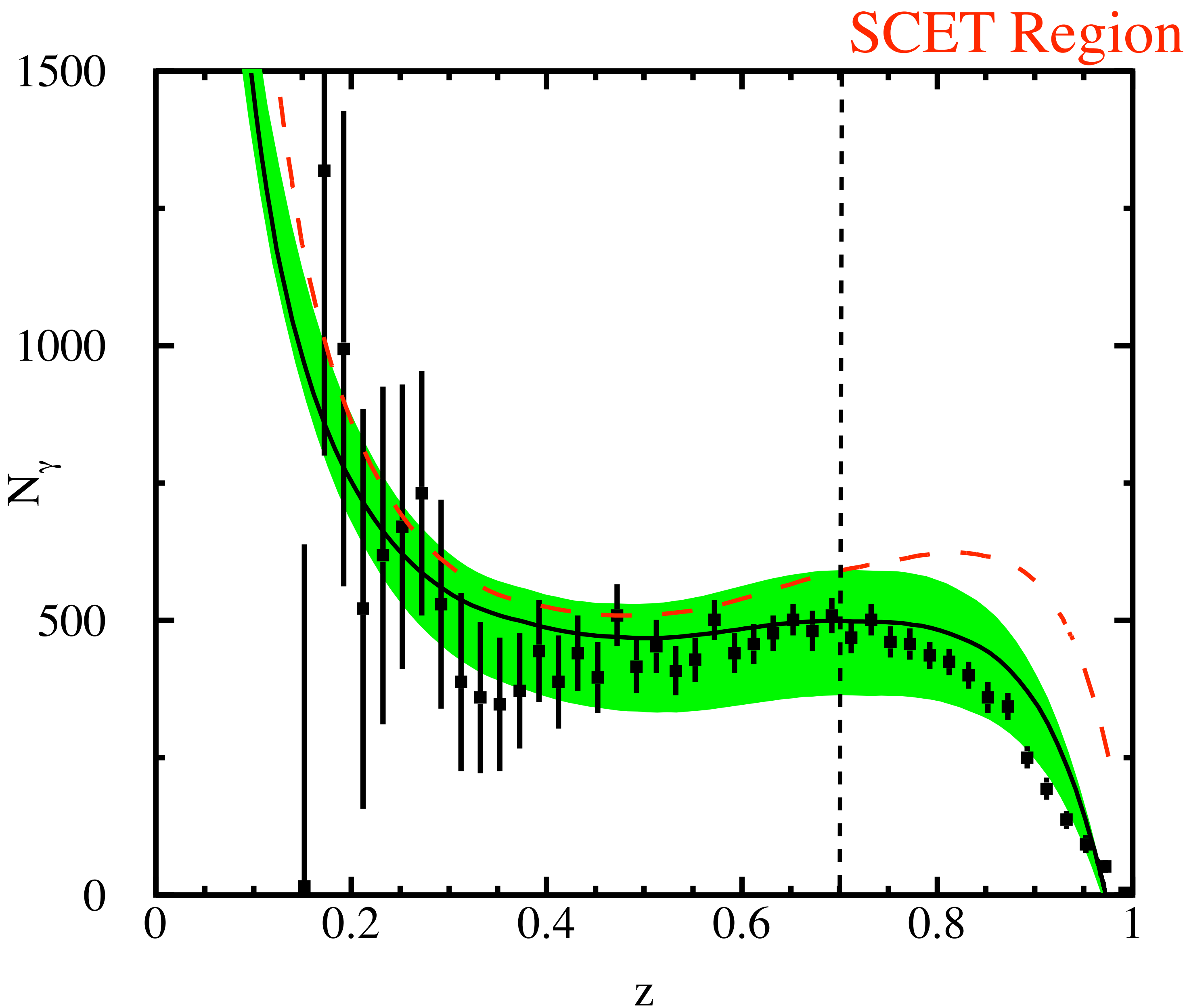
$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dX_\gamma} = \int d\xi\, S(\xi,\mu) J(\xi - X_\gamma, \mu)$$

- Color-Singlet Contribution:

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dX_\gamma} = \Theta(M_\Upsilon - 2X_\gamma m_b)\frac{8X_\gamma}{9} J_1(X_\gamma)$$

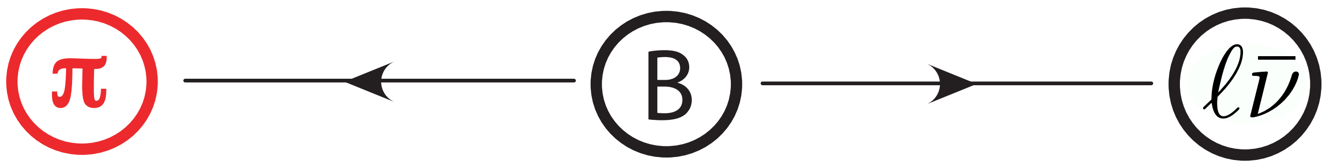
Radiative Υ Decay

- Comparison to CLEO data:



$$B \longrightarrow \pi + \ell\bar{\nu}$$

Factorization awry



- $\mathrm{SCET_I}$ gives a factorized form

  Recall $\mathrm{SCET_I}$ is appropriate for energetic jets

$$f(E) = \int dz\, T(z,E)\, \zeta_J^{BM}(z,E) + C(E)\, \zeta^{BM}(E)$$

- Simply further using $\mathrm{SCET_{II}}$

$$\zeta_J^{BM}(z) = f_M f_B \int_0^1 dx \int_0^\infty dk^+ J(z,x,k^+,E)\phi_M(x)\phi_B(k^+)$$

$\zeta^{BM} = \,?$ Endpoint divergences prevent further simplification

Summary & Conclusions

- Flavor of Soft Collinear Effective Theory: light-like particles interacting with a soft background
  - Derive factorization
  - Sum logarithms
  - Systematically treat power corrections
- Scope of applications is large
  - Color-suppressed $B \to D\pi$ decays
  - Radiative decays of the Υ
- Mystery: factorization of soft form-factor in $B \to \pi + \ell\bar{\nu}$
- Direction: control of non-perturbative physics in hadronic collisions
- Only scratched the surface: so much left to do...